تمارين (1-1)

س1/ ضع كلا مما يأتي بالصيغة العادية للعدد المركب :



$$i^5 = i^4 \times i = 0 + i$$

$$i^6 = i^4 \times i^2 = -1 + 0i$$

$$i^{124} = (i^4)^{31} = 1 + 0i$$

$$i^{999} = (i^4)^{249} \times i^3 = 0 - i$$

$$i^{4n+1} = (i^4)^n \times i = 0 + i$$

$$(2+3i)^2 + (12+2i) = (4 + 12i - 9) + (12 + 2i)$$

$$= (-5 + 12i) + (12 + 2i) = 7 + 14i$$

$$(1+i)^4 - (1-i)^4 = ((1+i)^2)^2 - ((1-i)^2)^2$$

$$= (1 + 2i - 1)^2 - (1 - 2i - 1)^2 = (2i)^2 - (-2i)^2$$

$$= -4 + 4 = 0 + 0i$$

$$\frac{12+i}{i} \times \frac{-i}{-i} = \frac{-12i - i^2}{1} = \frac{1 - 12i}{1} = 1 - 12i$$

$$\frac{3+4i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{9 + 24i - 16}{9 + 16} = \frac{-7 + 24i}{25} = \frac{-7}{25} + \frac{24}{25}i$$

$$\frac{i}{2+3i}\times\frac{2-3i}{2-3i}=\frac{2i-3i^2}{4+9}=\frac{3+2i}{13}=\frac{3}{13}+\frac{2}{13}i$$

$$\left(\frac{3+i}{1+i}\right)^3=\left(\frac{3+i}{1+i}\times\frac{1-i}{1-i}\right)^3=\left(\frac{3-3i+i-i^2}{1+1}\right)^3=\left(\frac{4-2i}{2}\right)^3$$

$$=\left(\frac{4}{2}-\frac{2i}{2}\right)^3=(2-i)^3=(2-i)^2(2-i)$$

$$=(4-4i-1)(2-i)=(3-4i)(2-i)$$

$$=(6-3i-8i-4)=2-11i$$

$$\frac{2+3i}{1-i}\times\frac{1+4i}{4+i}=\frac{2+8i+3i-12}{4+i-4i+1}=\frac{-10+11i}{5-3i}\times\frac{5+3i}{5+3i}$$

$$=\frac{-50-30i+55i-33}{25+9}=\frac{-83+25i}{34}=\frac{-83}{34}+\frac{25}{34}i$$

$$(1+i)^3+(1-i)^3=(1+i)^2(1+i)+(1+i)^2(1+i)$$

$$=(1+2i-1)(1+i)+(1-2i-1)(1-i)$$

$$=2i(1+i)+(-2i)(1-i)=2i-2-2i-2$$

$$=-4+0i$$

س2/ جد قيمة كل من x, y الحقيقيتين اللتين تحققان المعادلات الآتية :

$$y + 5i = (2x + i)(x + 2i)$$

$$y + 5i = 2x^2 + 4xi + xi - 2$$

$$2x^2 - 2 = y \qquad \dots (1)$$

$$5x = 5 \Rightarrow x = 1 \qquad \dots (2)$$

نعوض معادلة 2 في 1

$$2(1)^2 - 2 = y \Rightarrow y = 0$$

$$8i = (x + 2i)(y + 2i) + 1$$

$$8i = xy + 2xi + 2yi - 4 + 1$$

$$8i = xy + (2x + 2y)i - 3$$

$$xy - 3 = 0 \Rightarrow x = \frac{3}{y} \qquad \dots (1)$$

$$2x + 2y = 8 \Rightarrow x = 4 - y \qquad \dots (2)$$

نعوض معادلة 1 في 2 نحصل على :

$$\frac{3}{y} = 4 - y \Rightarrow 4y - y^2 = 3 \Rightarrow y^2 - 4y + 3 = 0$$

$$(y - 3)(y - 1) = 0$$

Either $y - 3 = 0 \Rightarrow y = 3 \Rightarrow x = \frac{3}{3} = 1$

Or $y - 1 = 0 \Rightarrow y = 1 \Rightarrow x = \frac{3}{1} = 3$



c) $\left(\frac{1-i}{1+i}\right)+(x+yi)=(1+2i)^2$

$$\Rightarrow\left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right)+(x+yi)=(1+4i-4)$$

$$\Rightarrow\left(\frac{1-i-i-1}{1+1}\right)+(x+yi)=(-3+4i)$$

$$\Rightarrow\left(\frac{-2i}{2}\right)+(x+yi)=(-3+4i)\Rightarrow -i+x+yi=-3+4i$$

$$\Rightarrow x=-3$$

$$\Rightarrow -1+y=4\Rightarrow y=5$$



d) $\frac{2-i}{1+i}x+\frac{3-i}{2+i}y=\frac{1}{i}$

$$\Rightarrow\left(\frac{2-i}{1+i}\times\frac{1-i}{1-i}\right)x+\left(\frac{3-i}{2+i}\times\frac{2-i}{2-i}\right)y=\frac{1}{i}\times\frac{-i}{-i}$$

$$\Rightarrow\frac{2-2i-i-1}{1+1}x+\frac{6-3i-2i-1}{4+1}y=\frac{-i}{1}$$

$$\Rightarrow\frac{1-3i}{2}x+\frac{5-5i}{5}y=0-i\Rightarrow\frac{1}{2}x-\frac{3}{2}xi+y-yi=0-i$$

$\Rightarrow\frac{1}{2}x+y=0$　　　نضرب طرفي المعادلة ب2

$\Rightarrow x+2y=0\Rightarrow x=-2y$　　... (1)



$\Rightarrow -\frac{3}{2}x - y = -1$ نضرب طرفي المعادلة ب-2 ...

$\Rightarrow 3x + 2y = 2$

$\Rightarrow x = \dfrac{2-2y}{3} \quad \dots (2)$

نعوض معادلة (1) بمعادلة (2) نحصل على :

$-2y = \dfrac{2-2y}{3} \Rightarrow -6y + 2y = 2 \Rightarrow -4y = 2 \Rightarrow y = -\dfrac{1}{2}$

$x = -2\left(-\dfrac{1}{2}\right) \Rightarrow x = 1$

س3/ اثبت أن :

$a)\quad \dfrac{1}{(2-i)^2} - \dfrac{1}{(2+i)^2} = \dfrac{8}{25}i$

$L.S \Rightarrow \dfrac{1}{(4-4i-1)} - \dfrac{1}{(4+4i-1)} = \dfrac{1}{3-4i} - \dfrac{1}{3+4i}$

$= \left(\dfrac{1}{3-4i} \times \dfrac{3+4i}{3+4i}\right) - \left(\dfrac{1}{3+4i} \times \dfrac{3-4i}{3-4i}\right) = \dfrac{3+4i}{9+16} - \dfrac{3-4i}{9+16}$

$= \dfrac{3+4i}{25} + \dfrac{(-3+4i)}{25} = \dfrac{8}{25}i = R.S$

$b)\quad \dfrac{(1-i)^2}{1+i}+\dfrac{(1+i)^2}{1-i}=-2$

$L.S \Rightarrow \dfrac{(1-2i-1)}{1+i}+\dfrac{(1+2i-1)}{1-i}$

$=\left(\dfrac{-2i}{1+i}\times\dfrac{1-i}{1-i}\right)+\left(\dfrac{2i}{1-i}\times\dfrac{1+i}{1+i}\right)=\left(\dfrac{-2i-2}{2}\right)+\left(\dfrac{2i-2}{2}\right)$

$=-i-1+i-1=-2=R.S$

$c)\quad (1-i)(1-i^2)(1-i^3)=4$

$L.S \Rightarrow (1-i)\big(1-(-1)\big)\big(1-(-i)\big)=(1-i)(2)(1+i)$

$=2(1+1)=2(2)=4=R.S$

س4/ حلل كلا من الاعداد $29 , 125 , 41 , 85$ الى حاصل ضرب عاملين من الصورة $a+bi$ حيث a, b عددان نسبيان.

$29=(25+4)=(25-4i^2)=(5-2i)(5+2i)$

$125=(100+25)=(100-25i^2)=(10-5i)(10+5i)$

$41=(25+16)=(25-16i^2)=(5-4i)(5+4i)$

$85=(81+4)=(81-4i^2)=(9-2i)(9+2i)$



س5/ جد قيمة x , y الحقيقيتين اذا علمت ان $\frac{6}{x+yi}$, $\frac{3+i}{2-i}$ مترافقان.

الحل :

$$\overline{\left(\frac{3+i}{2-i}\right)} = \left(\frac{6}{x+yi}\right) \Rightarrow \left(\frac{3-i}{2+i} \times \frac{2-i}{2-i}\right) = \left(\frac{6}{x+yi}\right)$$

$$\Rightarrow \frac{6-3i-2i-1}{4+1} = \frac{6}{x+yi} \Rightarrow \frac{5-5i}{5} = \frac{6}{x+yi}$$

$$\Rightarrow 1-i = \frac{6}{x+yi} \Rightarrow x+yi = \frac{6}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow x+yi = \frac{6+6i}{1+1} \Rightarrow x+yi = 3+3i$$

$$x = 3 \quad , \quad y = 3$$

